1. Explain the prediction of range performance.

The simple form of the radar equation expressed the maximum radar range \( R_{\text{max}} \), in terms of radar and target parameters:

\[
R_{\text{max}} = \left( \frac{P_t G A_e \sigma}{4\pi^2 S_{\text{min}}} \right)^{1/4}
\]

where
- \( P_t \) = transmitted power, watts
- \( G \) = antenna gain
- \( A_e \) = antenna effective aperture, m\(^2\)
- \( \sigma \) = radar cross section, m\(^2\)
- \( S_{\text{min}} \) = minimum detectable signal, watts

All the parameters are to some extent under the control of the radar designer, except for the target cross section \( \sigma \). The radar equation states that if long ranges are desired, the transmitted power must be large, the radiated energy must be concentrated into a narrow beam (high transmitting antenna gain), the received echo energy must be collected with a large antenna aperture (also synonymous with high gain), and the receiver must be sensitive to weak signals.

In practice, however, the simple radar equation does not predict the range performance of actual radar equipments to a satisfactory degree of accuracy. The predicted values of radar range are usually optimistic. In some cases the actual range might be only half that predicted. Part of this discrepancy is due to the failure of Eq. above to explicitly include the various losses that can occur throughout the system or the loss in performance usually experienced when electronic equipment is operated in the field rather than under laboratory-type conditions. Another important factor that must be considered in the radar equation is the statistical or unpredictable nature of several of the parameters. The minimum detectable signal \( S_{\text{min}} \) and the target cross section \( \sigma \) are both statistical in nature and must be expressed in statistical terms.

Other statistical factors which do not appear explicitly in Eq. but which have an effect on the radar performance are the meteorological conditions along the propagation path and the performance of the radar operator, if one is employed. The statistical nature of these several parameters does not allow the maximum radar range to be described by a single number. Its specification must include a statement of the probability that the radar will detect a certain type of target at a particular range.
2. What is meant by minimum detectable signal in radar?

The ability of a radar receiver to detect a weak echo signal is limited by the noise energy that occupies the same portion of the frequency spectrum as does the signal energy. The weakest signal the receiver can detect is called the minimum detectable signal. The specification of the minimum detectable signal is sometimes difficult because of its statistical nature and because the criterion for deciding whether a target is present or not may not be too well defined.

Detection is based on establishing a threshold level at the output of the receiver. If the receiver output exceeds the threshold, a signal is assumed to be present. This is called threshold detection. Consider the output of a typical radar receiver as a function of time (Fig. 2.1). This might represent one sweep of the video output displayed on an A-scope. The envelope has a fluctuating appearance caused by the random nature of noise. If a large signal is present such as at A in Fig. 2.1, it is greater than the surrounding noise peaks and can be recognized on the basis of its amplitude. Thus, if the threshold level were set sufficiently high, the envelope would not generally exceed the threshold if noise alone were present, but would exceed it if a strong signal were present. If the signal were small, however, it would be more difficult to recognize its presence. The threshold level must be low if weak signals are to be detected, but it cannot be so low that noise peaks cross the threshold and give a false indication of the presence of targets.

The voltage envelope of Fig. 2.1 is assumed to be from a matched-filter receiver. A matched filter is one designed to maximize the output peak signal to average noise (power) ratio. It has a frequency-response function which is proportional to the complex conjugate of the signal spectrum. (This is not the same as the concept of "impedance match" of circuit theory.) The ideal matched-filter receiver cannot always be exactly realized in practice, but it is possible to approach it with practical receiver circuits. A matched filter for a radar transmitting a rectangular-shaped pulse is usually characterized by a bandwidth B approximately the reciprocal of the pulse width $\tau$, or $B\tau \approx 1$. The output of a matched-filter receiver is
Fig. 2.1 Typical envelope of the radar receiver output as a function of time. A, and B and C represent signal plus noise. A and B would be valid detections, but C is a missed detection.

the cross-correlation between the received waveform and a replica of the transmitted waveform. Hence it does not preserve the shape of the input waveform. (There is no reason to wish to preserve the shape of the received waveform so long as the output signal-to-noise ratio is maximized.)

3. Discuss in detail the quantitative analysis of receiver noise and hence derive the expression for minimum detectable signal.

Receiver Noise:

Since noise is the chief factor limiting receiver sensitivity, it is necessary to obtain some means of describing it quantitatively. Noise is unwanted electromagnetic energy which interferes with the ability of the receiver to detect the wanted signal. It may originate within the receiver itself, or it may enter via the receiving antenna along with the desired signal. If the radar were to operate in a perfectly noise-free environment so that no external sources of noise accompanied the desired signal, and if the receiver itself were so perfect that it did not generate any excess noise, there would still exist an unavoidable component of noise generated by the thermal motion of the conduction electrons in the ohmic portions of the receiver input stages. This is called thermal noise, or Johnson noise, and is directly proportional to the temperature of the ohmic portions of the circuit and the receiver bandwidth. The available thermal-noise power generated by a receiver of bandwidth $B_n$ (in hertz) at a temperature $T$ (degrees Kelvin) is equal to
Available thermal-noise power = $kTB_n$

where $k$ = Boltzmann's constant = $1.38 \times 10^{-23}$ J/deg. If the temperature $T$ is taken to be 290 K, which corresponds approximately to room temperature ($62^\circ$F), the factor $kT$ is $4 \times 10^{-21}$ W/Hz of bandwidth. If the receiver circuitry were at some other temperature, the thermal-noise power would be correspondingly different.

A receiver with a reactance input such as a parametric amplifier need not have any significant ohmic loss. The limitation in this case is the thermal noise seen by the antenna and the ohmic losses in the transmission line.

For radar receivers of the superheterodyne type (the type of receiver used for most radar applications), the receiver bandwidth is approximately that of the intermediate-frequency stages. It should be cautioned that the bandwidth $B$, of Eq. is not the 3-dB, or half-power, bandwidth commonly employed by electronic engineers. It is an integrated bandwidth and is given by

$$B_n = \frac{\int_{-\infty}^{\infty} |H(f)|^2 \, df}{|H(f_0)|^2}$$

where $H(f)$ = frequency-response characteristic of IF amplifier (filter) and $f_0$ = frequency of maximum response (usually occurs at midband). When $H(f)$ is normalized to unity at midband (maximum-response frequency), $H(f_0) = 1$. The bandwidth $B_n$ is called the noise bandwidth and is the bandwidth of an equivalent rectangular filter whose noise-power output is the same as the filter with characteristic $H(f)$. The 3-dB bandwidth is defined as the separation in hertz between the points on the frequency-response characteristic where the response is reduced to 0.707 (3 dB) from its maximum value. The 3-dB bandwidth is widely used, since it is easy to measure. The measurement of noise bandwidth however, involves a complete knowledge of the response characteristic $H(f)$. The frequency-response characteristics of many practical radar receivers are such that the 3-dB and the noise bandwidths do not differ appreciably. Therefore the 3-dB bandwidth may be used in many cases as an approximation to the noise bandwidth.

The noise power in practical receivers is often greater than can be accounted for by thermal noise alone. The additional noise components are due to mechanisms other than the thermal agitation of the conduction electrons. The exact origin of the extra noise components is not important except to know that it exists. No matter whether the noise is generated by a thermal mechanism or by some other mechanism, the total noise at the output of the receiver may be considered to be equal to the thermal-noise power obtained from an "ideal"
receiver multiplied by a factor called the noise figure. The noise figure $F_n$ of a receiver is defined by the equation

$$F_n = \frac{N_o}{kT_o B_n G_a} = \frac{\text{noise out of practical receiver}}{\text{noise out of ideal receiver at std temp } T_0}$$

where $N_o = \text{noise output from receiver}$, and $G_a = \text{available gain}$. The standard temperature $T$ is taken to be 290 K, according to the Institute of Electrical and Electronics Engineers definition. The noise $N_o$ is measured over the linear portion of the receiver input-output characteristic, usually at the output of the IF amplifier before the nonlinear second detector. The receiver bandwidth $B_n$ is that of the IF amplifier in most receivers. The available gain $G_a$ is the ratio of the signal out $S_o$ to the signal in $S_i$, and $kT_o B_n$ is the input noise $N_i$ in an ideal receiver. Equation above may be rewritten as

$$F_n = \frac{S_o}{N_o}$$

The noise figure may be interpreted, therefore, as a measure of the degradation of signal-to-noise-ratio as the signal passes through the receiver.

Rearranging Eq. above the input signal may be expressed as

$$S_i = \frac{kT_o B_n F_n S_o}{N_o}$$

If the minimum detectable signal $S_{\text{min}}$, is that value of $S_i$ corresponding to the minimum ratio of output (IF) signal-to-noise ratio $(S_o/N_o)_{\text{min}}$ necessary for detection, then

$$S_{\text{min}} = kT_o B_n F_n \left(\frac{S_o}{N_o}\right)_{\text{min}}$$

Substituting Eq. discussed above into Eq. earlier results in the following form of the radar equation:

$$\frac{R_{\text{max}}^4}{R_{\text{max}}} = \frac{P_i G A_o \sigma}{(4\pi)^2 kT_o B_n F_n (S_o/N_o)_{\text{min}}}$$

GRIET-ECE
4. Describe how threshold level for detection is decided in the presence of receiver noise for a specified probability of occurrence of false alarms by applying statistical noise theory.

Consider an IF amplifier with bandwidth $B_{IF}$ followed by a second detector and a video amplifier with bandwidth $B_v$, (Fig. 4.1). The second detector and video amplifier are assumed to form an envelope detector, that is, one which rejects the carrier frequency but passes the modulation envelope. To extract the modulation envelope, the video bandwidth must be wide enough to pass the low-frequency components generated by the second detector, but not so wide as to pass the high-frequency components at or near the intermediate frequency. The video bandwidth $B_v$ must be greater than $B_{IF}/2$ in order to pass all the video modulation. Most radar receivers used in conjunction with an operator viewing a CRT display meet this condition and may be considered envelope detectors. Either a square-law or a linear detector may be assumed since the effect on the detection probability by assuming one instead of the other is usually small.

![Fig.4.1 Envelope detector](image)

The noise entering the IF filter (the terms filter and amplifier are used interchangeably) is assumed to be gaussian, with probability-density function given by

$$p(v) = \frac{1}{\sqrt{2\pi}\psi_o} \exp\left(-\frac{v^2}{2\psi_o}\right)$$

where $p(v)\, dv$ is the probability of finding the noise voltage $v$ between the values of $v$ and $v + dv$, $\psi_o$ is the variance, or mean-square value of the noise voltage, and the mean value of $v$ is taken to be zero. If gaussian noise were passed through a narrowband IF filter-one whose bandwidth is small compared with the mid frequency-the probability density of the envelope of the noise voltage output is shown by Rice to be

$$p(R) = \frac{R}{\psi_o} \exp\left(-\frac{R^2}{2\psi_o}\right)$$
where $R$ is the amplitude of the envelope of the filter output. Equation above is a form of the Rayleigh probability-density function. The probability that the envelope of the noise voltage will lie between the values of $V_1$ and $V_2$ is

$$
\text{Probability } (V_1 < R < V_2) = \int_{V_1}^{V_2} \frac{R}{\psi_o} \exp \left( -\frac{R^2}{2\psi_o} \right) dR
$$

The probability that the noise voltage envelope will exceed the voltage threshold $V_T$ is

$$
\text{Probability } (V_T < R < \infty) = \int_{V_T}^{\infty} \frac{R}{\psi_o} \exp \left( -\frac{R^2}{2\psi_o} \right) dR
$$

$$
= \exp \left( -\frac{V_T^2}{2\psi_o} \right) = P_{fa}
$$

Whenever the voltage envelope exceeds the threshold, target detection is considered to have occurred, by definition. Since the probability of a false alarm is the probability that noise will cross the threshold, Eq. above gives the probability of a false alarm, denoted $P_{fa}$

The average time interval between crossings of the threshold by noise alone is defined as the false-alarm time $T_{fa}$,

$$
T_{fa} = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} T_k
$$

where $T_k$ is the time between crossings of the threshold $V_T$ by the noise envelope, when the slope of the crossing is positive. The false-alarm probability may also be defined as the ratio of the duration of time the envelope is actually above the threshold to the total time it could have been above the threshold, or

$$
P_{fa} = \frac{\sum_{k=1}^{N} t_k}{\sum_{k=1}^{N} T_k} = \frac{\langle t_k \rangle \psi_o}{\langle T_k \rangle \psi_o} = \frac{1}{T_{fa} \psi_o}
$$
Fig. 4.1 Envelope of receiver output illustrating false alarms due to noise.

where \( t_k \) and \( T_k \) are defined in Fig. 4.1. The average duration of a noise pulse is approximately the reciprocal of the bandwidth \( B \), which in the case of the envelope detector is \( B_{IF} \). Equating equations discussed above we get

\[
T_{fa} = \frac{1}{B_{IF}} \exp \left( \frac{V^2}{2\Psi_o} \right)
\]

A plot of Eq. above is shown in Fig. 4.2, with \( V^2 / 2 \Psi_o \) as the abscissa. If, for example, the bandwidth of the IF amplifier were 1 MHz and the average false-alarm time that could be tolerated were 15 min, the probability of a false alarm is \( 1.11 \times 10^{-9} \). From Eq. above the threshold voltage necessary to achieve this false-alarm time is 6.45 times the rms value of the noise voltage.

The false-alarm probabilities of practical radars are quite small. The reason for this is that the false-alarm probability is the probability that a noise pulse will cross the threshold during an interval of time approximately equal to the reciprocal of the bandwidth. For a 1-MHz bandwidth, there are of the order of \( 10^6 \) noise pulses per second. Hence the false-alarm probability of any one pulse must be small (< \( 10^{-6} \)) if false-alarm times greater than 1 s are to be obtained.
Fig. 4.2 Average time between false alarms as a function of the threshold level $V_T$ and the receiver bandwidth $B$; $\Psi_o$ is the mean square noise voltage.

Consider sine-wave signals of amplitude $A$ to be present along with noise at the input to the IF filter. The frequency of the signal is the same as the IF midband frequency $f_{IF}$. The output or the envelope detector has a probability-density function given by

$$p_x(R) = \frac{R}{\Psi_o} \exp \left( -\frac{R^2 + A^2}{2\Psi_o} \right) I_0 \left( \frac{RA}{\Psi_o} \right)$$
where $I_0(Z)$ is the modified Bessel function of zero order and argument $Z$. For $Z$ large, an asymptotic expansion for $I_0(Z)$ is

$$I_0(Z) \approx \frac{e^Z}{\sqrt{2\pi Z}} \left(1 + \frac{1}{8Z} + \cdots \right)$$

When the signal is absent, $A = 0$, the probability-density function for noise alone. Equation above is sometimes called the Rice probability-density function. The probability that the signal will be detected (which is the probability of detection) is the same as the probability that the envelope $R$ will exceed the predetermined threshold $V_T$. The probability of detection $P_d$, is therefore

$$P_d = \left[ \int_{V_T}^{\infty} p_s(R) \, dR \right] = \int_{V_T}^{\infty} \frac{R}{\psi_o} \exp \left( - \frac{R^2 + A^2}{2\psi_o} \right) I_0 \left( \frac{RA}{\psi_o} \right) \, dR$$

This cannot be evaluated by simple means, and numerical techniques or a series approximation must be used. A series approximation valid when $RA/\Psi_o >> 1$, $A >> |R - A|$, and terms in $A^{-3}$ and beyond can be neglected is

$$P_d = \frac{1}{2} \left[ 1 - \text{erf} \left( \frac{V_T - A}{\sqrt{2\psi_o}} \right) \right] + \frac{\exp \left[ - (V_T - A)^2/2\psi_o \right]}{2\sqrt{2\pi} (A/\sqrt{\psi_o})} \times \left[ 1 + \frac{V_T - A}{4A} + \frac{(V_T - A)^2/\psi_o}{8A^2/\psi_o} + \cdots \right]$$

where the error function is defined as

$$\text{erf} Z = \frac{2}{\sqrt{\pi}} \int_0^Z e^{-u^2} \, du$$

Equation discussed above may be converted to power by replacing the signal to rms-noise-voltage ratio with the following:

$$\frac{A}{\sqrt{\psi_0}} = \frac{\text{signal amplitude}}{\text{rms noise voltage}} = \sqrt{2 (\text{rms signal voltage})/\text{rms noise voltage}} = \left( \frac{2 \text{ signal power}}{\text{noise power}} \right)^{1/2} = \left( \frac{2S}{N} \right)^{1/2}$$
5. Justify the requirement of integration of radar pulses to improve target detection process.

Many pulses are usually returned from any particular target on each radar scan and can be used to improve detection. The number of pulses \( n_B \) returned from a point target as the radar antenna scans through its beamwidth is

\[
\frac{\theta_B f_p}{\theta_s} = \frac{n_B f_p}{6 \omega_m}
\]

where
- \( \theta_B \) = antenna beamwidth, deg
- \( f_p \) = pulse repetition frequency, Hz
- \( \theta_s \) = antenna scanning rate, deg/s
- \( \omega_m \) = antenna scan rate, rpm

Typical parameters for a ground-based search radar might be pulse repetition frequency 300 Hz, 1.5° beamwidth, and antenna scan rate 5 rpm (30°/s). These parameters result in 15 hits from a point target on each scan. The process of summing all the radar echo pulses for the purpose of improving detection is called integration. Many techniques might be employed for accomplishing integration. All practical integration techniques employ some sort of storage device. Perhaps the most common radar integration method is the cathode-ray-tube display combined with the integrating properties of the eye and brain of the radar operator.

Integration may be accomplished in the radar receiver either before the second detector (in the IF) or after the second detector (in the video). A definite distinction must be made between these two cases. Integration before the detector is called predetection, or coherent, integration, while integration after the detector is called postdetection, or noncoherent, integration. Predetection integration requires that the phase of the echo signal be preserved if full benefit is to be obtained from the summing process. On the other hand, phase information is destroyed by the second detector; hence postdetection integration is not concerned with preserving RF phase. For this convenience, postdetection integration is not as efficient as predetection integration.

If \( n \) pulses, all of the same signal-to-noise ratio, were integrated by an ideal predetection integrator, the resultant, or integrated, signal-to-noise (power) ratio would be exactly \( n \) times that of a single pulse. If the same \( n \) pulses were integrated by an ideal postdetection device, the resultant signal-to-noise ratio would be less than \( n \) times that of a single pulse. This loss in
integration efficiency is caused by the nonlinear action of the second detector, which converts some of the signal energy to noise energy in the rectification process.

The comparison of predetection and postdetection integration may be briefly summarized by stating that although postdetection integration is not as efficient as predetection integration, it is easier to implement in most applications. Post detection integration is therefore preferred, even though the integrated signal-to-noise ratio may not be as great. An alert, trained operator viewing a properly designed cathode-ray tube display is a close approximation to the theoretical postdetection integrator.

The efficiency of postdetection integration relative to ideal predetection integration has been computed by Marcum when all pulses are of equal amplitude. The integration efficiency may be defined as follows:

\[ E_i(n) = \frac{(S/N)_1}{n(S/N)_n} \]

where \( n \) = number of pulses integrated
\((S/N)_1 \) = value of signal-to-noise ratio of a single pulse required to produce given probability of detection (for \( n = 1 \))
\((S/N)_n \) = value of signal-to-noise ratio per pulse required to produce same probability of detection when \( n \) pulses are integrated

6. Write short notes on radar cross section of targets.

Radar Cross Section of Targets:

The radar cross section of a target is the (fictional) area intercepting that amount of power which when scattered equally in all directions, produces an echo at the radar equal to that from the target; or in other terms,

\[ \sigma = \frac{\text{power reflected toward source/unit solid angle}}{\frac{\text{incident power density}}{4\pi}} = \lim_{R \to \infty} 4\pi R^2 \left| \frac{E_r}{E_i} \right|^2 \]
This equation is equivalent to the radar range equation. For most common types of radar targets such as aircraft, ships, and terrain, the radar cross section does not necessarily bear a simple relationship to the physical area, except that the larger the target size, the larger the cross section is likely to be.

Scattering and diffraction are variations of the same physical process. When an object scatters an electromagnetic wave, the scattered field is defined as the difference between the total field in the presence of the object and the field that would exist if the object were absent (but with the sources unchanged). On the other hand, the diffracted field is the total field in the presence of the object. With radar backscatter, the two fields are the same, and one may talk about scattering and diffraction interchangeably.

In theory, the scattered field, and hence the radar cross section, can be determined by solving Maxwell’s equations with the proper boundary conditions applied. Unfortunately, the determination of the radar cross section with Maxwell’s equations can be accomplished only for the most simple of shapes, and solutions valid over a large range of frequencies are not easy to obtain. The radar cross section of a simple sphere is shown in Fig.6.1 as a function of its circumference measured in wavelengths \(2\pi a/\lambda\), where \(a\) is the radius of the sphere and \(\lambda\) is the wavelength. The region where the size of the sphere is small compared with the wavelength \(2\pi a/\lambda \ll 1\) is called the Rayleigh region, after Lord Rayleigh who, in the early 1870’s first studied scattering by small particles. Lord Rayleigh was interested in the scattering of light by microscopic particles, rather than in radar. His work preceded the original electromagnetic echo experiments of Hertz by about fifteen years.

The Rayleigh scattering region is of interest to the radar engineer because the cross sections of raindrops and other meteorological particles fall within this region at the usual radar frequencies. Since the cross section of objects within the Rayleigh region varies as \(\lambda^{-4}\), rain and clouds are essentially invisible to radars which operate at relatively long wavelengths (low frequencies). The usual radar targets are much larger than raindrops or cloud particles, and lowering the radar frequency to the point where rain or cloud echoes are negligibly small will not seriously reduce the cross section of the larger desired targets. On the other hand, if it were desired to actually observe, rather than eliminate, raindrop echoes, as in a meteorological or weather-observing radar, the higher radar frequencies would be preferred.
At the other extreme from the Rayleigh region is the optical region, where the dimensions of the sphere are large compared with the wavelength \(2\pi a/\lambda >> 1\). For large \(2\pi a/\lambda\), the radar cross section approaches the optical cross section \(\pi a^2\). In between the optical and the Rayleigh region is the Mie, or resonance, region. The cross section is oscillatory with frequency within this region. The maximum value is 5.6 dB greater than the optical value, while the value of the first null is 5.5 dB below the optical value. (The theoretical values of the maxima and minima may vary according to the method of calculation employed.) The behavior of the radar cross sections of other simple reflecting objects as a function of frequency is similar to that of the sphere.

7. Discuss in detail about the Cross-section fluctuation.

Cross-section fluctuations:
In the minimum signal-to-noise ratio it is assumed that the echo signal received from a particular target did not vary with time. In practice, however, the echo signal from a target in motion is almost never constant. Variations in the echo signal may be caused by meteorological conditions, the lobe structure of the antenna pattern, equipment instabilities, or variations in the target cross section. The cross sections of complex targets (the usual type of radar target) are quite sensitive to aspect. Therefore, as the target aspect changes relative to the radar, variations in the echo signal will result.

One method of accounting for a fluctuating cross section in the radar equation is to select a lower bound, that is, a value of cross section that is exceeded some specified (large) fraction of time. The fraction of time that the actual cross section exceeds the selected value would be close to unity (0.95 or 0.99 being typical). For all practical purposes the value selected is a minimum and the target will always present a cross section greater than that selected. This procedure results in a conservative prediction of radar range and has the advantage of simplicity. The minimum cross section of typical aircraft or missile targets generally occurs at or near the head-on aspect.

However, to properly account for target cross-section fluctuations, the probability-density function and the correlation properties with time must be known for the particular target and type of trajectory. Curves of cross section as a function of aspect and knowledge of the trajectory with respect to the radar are needed to obtain a true description of the dynamical variations of cross section. The probability-density function gives the probability of finding any particular value of target cross section between the values of $\sigma$ and $\sigma + d\sigma$, while the autocorrelation function describes the degree of correlation of the cross section with time or number of pulses. The spectral density of the cross section (from which the autocorrelation function can be derived) is also sometimes of importance, especially in tracking radars. It is usually not practical to obtain the experimental data necessary to compute the probability-density function and the autocorrelation function from which the overall radar performance is determined. Most radar situations are of too complex a nature to warrant obtaining complete data. A more economical method to assess the effects of a fluctuating cross section is to postulate a reasonable model for the fluctuations and to analyze it mathematically. Swerling has calculated the detection probabilities for four different fluctuation models of cross section. In two of the four cases, it is assumed that the fluctuations are completely correlated during a particular scan but are completely uncorrelated from scan to scan. In the other two cases, the fluctuations are assumed to be more rapid and uncorrelated pulse to pulse. The four fluctuation models are as follows:

**Case 1:** The echo pulses received from a target on any one scan are of constant amplitude throughout the entire scan but are independent (uncorrelated) from scan to scan. This assumption ignores the effect of the antenna beam shape on the echo amplitude. An echo fluctuation of this
type will be referred to as scan-to-scan fluctuation. The probability-density function for the cross section $\sigma$ is given by the density function

$$p(\sigma) = \frac{1}{\sigma_\text{av}} \exp \left( - \frac{\sigma}{\sigma_\text{av}} \right) \quad \sigma \geq 0$$

where, $\sigma_\text{av}$ is the average cross-section over all target fluctuations.

**Case 2:** The probability-density function for the target cross section is also given by Eq. above but the fluctuations is more rapid than in case 1 and are taken to be independent from pulse to pulse instead of scan to scan.

**Case 3:** In this case the fluctuation is assumed to be independent from scan to scan as in case 1 but the probability-density function is given by

$$p(\sigma) = \frac{4\sigma}{\sigma_\text{av}^2} \exp \left( - \frac{2\sigma}{\sigma_\text{av}} \right)$$

**Case 4:** The fluctuation is pulse to pulse according to Eq. above.

The probability-density function assumed in cases 1 and 2 applies to a complex target consisting of many independent scatterers of approximately equal echoing areas. Although, in theory, the number of independent scatterers must be essentially infinite, in practice the number may be as few as four or five. The probability-density function assumed in cases 3 arid 4 is more indicative of targets that can be represented as one large reflector together with other small reflectors. In all the above cases, the value of cross section to be substituted in the radar equation is the average cross section $\sigma_\text{av}$. The signal-to-noise ratio needed to achieve a specified probability of detection without exceeding a specified false-alarm probability can be calculated for each model of target behavior. For purposes of comparison, the non fluctuating cross section will be called case 5.

8. Define and explain Transmitted power in radar equation and expressed the Radar equation in terms of the energy contained in the transmitted waveform.

**Transmitter Power:**

The power $P_t$ in the radar equation is called by the radar engineer the peak power. The peak pulse power as used in the radar equation is not the instantaneous peak power of a sine wave. It is
defined as the power averaged over that carrier-frequency cycle which occurs at the maximum of the pulse of power. (Peak power is usually equal to one-half the maximum instantaneous power.) The average radar power $P_{av}$ is also of interest in radar and is defined as the average transmitter power over the pulse-repetition period. If the transmitted waveform is a train of rectangular pulses of width $\tau$ and pulse-repetition period $T_p = 1/f_p$, the average power is related to the peak power by

$$P_{av} = \frac{P_t \tau}{T_p} = P_t \tau f_p$$

The ratio $P_{av}/P_t$, $\tau/T_p$, or $\tau f_p$ is called the duty cycle of the radar. A Pulse radar for detection of aircraft might have typically a duty cycle of 0.001, while a CW radar which transmits continuously has a duty cycle of unity. Writing the radar equation in terms of the average power rather than the peak power, we get

$$R_{max}^4 = \frac{P_{av} G A_e \sigma n E_0(n)}{(4\pi)^2 k T_0 F_n(B_n \tau)(S/N) f_p}$$

The bandwidth and the pulse width are grouped together since the product of the two is usually of the order of unity in most pulse-radar applications. If the transmitted waveform is not a rectangular pulse, it is sometimes more convenient to express the radar equation in terms of the energy $E_t = P_{av}/f_p$ contained in the transmitted waveform:

$$R_{max}^4 = \frac{E_t G A_e \sigma n E_0(n)}{(4\pi)^2 k T_0 F_n(B_n \tau)(S/N) f_p}$$

In this form, the range does not depend explicitly on either the wavelength or the pulse repetition frequency. The important parameters affecting range are the total transmitted energy $nE_t$, the transmitting gain $G$, the effective receiving aperture $A_e$, and the receiver noise figure $F_n$.

9. Write explanatory notes on pulse repetition frequency and range ambiguities.
The pulse repetition frequency (prf) is determined primarily by the maximum range at which targets are expected. If the prf is made too high the likelihood of obtaining target echoes from the wrong pulse transmission is increased. Echo signals received after an interval exceeding the pulse-repetition period are called multiple-time-around echoes. They can result in erroneous or confusing range measurements. Consider the three targets labeled A, B, and C in Fig. 9 a. Target A is located within the maximum unambiguous range $R_{\text{unamb}}$ of the radar, target B is at a distance greater than $R_{\text{unamb}}$ but less than $2R_{\text{unamb}}$ while target C is greater than $2R_{\text{unamb}}$ but less than $3R_{\text{unamb}}$. The appearance of the three targets on an A-scope is sketched in Fig. 9 b. The multiple-time-around echoes on the A-scope cannot be distinguished from proper target echoes actually within the maximum unambiguous range. Only the range measured for target A is correct; those for Band C are not.

One method of distinguishing multiple-time-around echoes from unambiguous echoes is to operate with a varying pulse repetition frequency. The echo signal from an unambiguous range target will appear at the same place on the A-scope on each sweep no matter whether the prf is modulated or not. However, echoes from multiple-time-around targets will be spread over a finite range as shown in Fig. 9 c. The prf may be changed continuously within prescribed limits or it may be changed discretely among several predetermined values. The number of separate pulse repetition frequencies will depend upon the degree of the multiple-time targets. Second-time targets need only two separate repetition frequencies in order to be resolved.

Fig.9 Multiple-time-around echoes that give rise to ambiguities in range (a) Three targets A, B and C, where A is within $R_{\text{unamb}}$, and B and C are multiple-time-around targets;
(b) the appearance of the three targets on the A-scope; (c) appearance of the three targets on the A-scope with a changing prf.

Instead of modulating the prf, other schemes that might he employed to" mark" successive pulses so as to identify multiple-time-around echoes include changing the pulse: amplitude, pulse width, frequency, phase, or polarization of transmission from pulse to pulse. Generally, such schemes are not so successful in practice as one would like. One of the fundamental limitations is the fold over of nearby targets; that is, nearby strong ground targets (clutter) can be quite large and can mask weak multiple-time-around targets appearing at the same place on the display. Also, more time is required to process the data when resolving ambiguities.

Ambiguities may theoretically be resolved by observing the variation of the echo signal with time (range). This is not always a practical technique; however, since the echo-signal amplitude can fluctuate strongly for reasons other than a change in range. Instead, the range ambiguities in a multiple prf radar can be conveniently decoded and the true range found by the use of the Chinese remainder theorem or other computational algorithms.

10. Write an explanatory notes on system losses.

System Losses:

One of the important factors omitted from the simple radar equation was the losses that occur throughout the radar system. The losses reduce the signal-to-noise ratio at the receiver output. They may be of two kinds, depending upon whether or not they can be predicted with any degree of precision beforehand.

The antenna beam-shape loss, collapsing loss, and losses in the microwave plumbing are examples of losses which can be calculated if the system configuration is known. These losses are very real and cannot be ignored in any serious prediction of radar performance. Losses not readily subject to calculation and which are less predictable include those due to field degradation and to operator fatigue or lack of operator motivation. Estimates of the latter type of loss must be made on the basis of prior experience and experimental observations. They may be subject to considerable variation and uncertainty. Although the loss associated with any one factor may be small, there are many possible loss mechanisms in a complete radar system, and their sum total can be significant.
In this section loss (number greater than unity) and efficiency (number less than unity) are used interchangeably. One is simply the reciprocal of the other.

**Plumbing loss:** There is always some finite loss experienced in the transmission lines which connect the output of the transmitter to the antenna. The losses in decibels per 100 ft for radar transmission lines are shown in Fig.10. At the lower radar frequencies the transmission line introduces little loss, unless its length is exceptionally long. At the higher radar frequencies, attenuation may not always be small and may have to be taken into account. In addition to the losses in the transmission line itself, an additional loss can occur at each connection or bend in the line and at the antenna rotary joint if used. Connector losses are usually small, but if the connection is poorly made, it can contribute significant attenuation. Since the same transmission line is generally used for both receiving and transmission, the loss to be inserted in the radar equation is twice the one-way loss.

The signal suffers attenuation as it passes through the duplexer. Generally, the greater the isolation required from the duplexer on transmission, the larger will be the insertion loss. By insertion loss is meant the loss introduced when the component, in this case the duplexer, is inserted into the transmission line. The precise value of the insertion loss depends to a large extent on the particular design. For a typical duplexer it might be of the order of 1 dB. A gas-tube duplexer also introduces loss when in the fired condition (arc loss); approximately 1 dB is typical.

In S-band (3000 MHz) radar, for example, the plumbing losses might be as follows:

<table>
<thead>
<tr>
<th>Description</th>
<th>Loss (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 ft of RG-113/U A1 waveguide transmission line (two-way)</td>
<td>1.0</td>
</tr>
<tr>
<td>Loss due to poor connections (estimate)</td>
<td>0.5</td>
</tr>
<tr>
<td>Rotary-joint loss</td>
<td>0.4</td>
</tr>
<tr>
<td>Duplexer loss</td>
<td>1.5</td>
</tr>
<tr>
<td>Total plumbing loss</td>
<td>3.4</td>
</tr>
</tbody>
</table>
Beam-shape loss: The antenna gain that appears in the radar equation was assumed to be a constant equal to the maximum value. But in reality the train of pulses returned from a target with a scanning radar is modulated in amplitude by the shape of the antenna beam. To properly take into account the pulse-train modulation caused by the beam shape, the computations of the probability of detection would have to be performed assuming a modulated train of pulses rather than constant-amplitude pulses. Some authors do indeed take account of the beam shape in this manner when computing the probability of detection. Therefore, when using published computations of probability of detection it should be noted whether the effect of the beam shape has been included. This approach is not used. Instead a beam-shape loss is added to the radar equation to account for the fact that the maximum gain is employed in the radar equation rather than a gain that changes pulse to pulse. This is a simpler, albeit less accurate, method. It is based on calculating the reduction in signal power and thus does not depend on the probability of detection. It applies for detection probabilities in the vicinity of 0.50, but it is used as an approximation with other values as a matter of convenience.
Let the one-way-power antenna pattern be approximated by the gaussian expression \( \exp(-2.78 \theta^2 / \theta_B^2) \), where \( \theta \) is the angle measured from the center of the beam and \( \theta_B \) is the beamwidth measured between half-power points. If \( n_B \) is the number of pulses received within the half-power beamwidth \( \theta_B \), and \( n \) is the total number of pulses integrated (\( n \) does not necessarily equal \( n_B \)), then the beam-shape loss (number greater than unity) relative to a radar that integrates all \( n \) pulses with an antenna gain corresponding to the maximum gain at the beam center is

\[
\text{Beam-shape loss} = \frac{n}{1 + 2 \sum_{k=1}^{\lfloor n/2 \rfloor} \exp \left( -5.55 k^2 / n_B^2 \right)}
\]

**Limiting loss:** Limiting in the radar receiver can lower the probability of detection. Although a well-designed and engineered receiver will not limit the received signal under normal circumstances, intensity modulated CRT displays such as the PPI or the B-scope have limited dynamic range and may limit. Some receivers, however, might employ limiting for some special purpose, as for pulse compression processing for example.

Limiting results in a loss of only a fraction of a decibel for a large number of pulses integrated provided the limiting ratio (ratio of video limit level to rms noise level) is as large as 2 or 3. Other analyses of band pass limiters show that for small signal-to-noise ratio, the reduction in the signal-to-noise ratio of a sine-wave imbedded in narrowband gaussian noise is \( \pi/4 \) (about 1 dB). However, by appropriately shaping the spectrum of the input noise, it has been suggested that the degradation can be made negligibly small.

**Collapsing loss:** If the radar were to integrate additional noise samples along with the wanted signal-to-noise pulses, the added noise results in degradation called the collapsing loss. It can occur in displays which collapse the range information, such as the C-scope which displays elevation vs. azimuth angle. The echo signal from a particular range interval must compete in a collapsed-range C-scope display, not only with the noise energy contained within that range interval but with the noise energy from all other range intervals at the same elevation and azimuth. In some 3D radars (range, azimuth, and elevation) that display the outputs at all elevations on a single PPI (range, azimuth) display, the collapsing of the 3D radar information onto a 2D display results in a loss. A collapsing loss can occur when the output of a high-resolution radar is displayed on a device whose resolution is coarser than that inherent in the radar. A collapsing loss also results if the outputs of two (or more) radar receivers are combined and only one contains signal while the other contains noise.
The mathematical derivation of the collapsing loss, assuming a square-law detector may be carried out as suggested by Marcum who has shown that the integration of m noise pulses, along with n signal-plus-noise pulses with signal-to-noise ratio per pulse $(S/N)_n$, is equivalent to the integration of $m + n$ signal-to-noise pulses each with signal-to-noise ratio $n(S/N)_n/(m + n)$. The collapsing loss in this case is equal to the ratio of the integration loss $L_i$ for $m + n$ pulses to the integration loss for $n$ pulses, or

$$L_i(m, n) = L_i(m + n)$$

$$L_i(n)$$